

## W100/301

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NATIONAL  
QUALIFICATIONS  
2002

FRIDAY, 18 JANUARY  
9.00 AM – 10.10 AM

MATHEMATICS  
HIGHER

Units 1, 2 and 3

Paper 1

(Non-calculator)

### Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

**FORMULAE LIST**

**Circle:**

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2 \cos^2 A - 1$

$= 1 - 2 \sin^2 A$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals:**

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

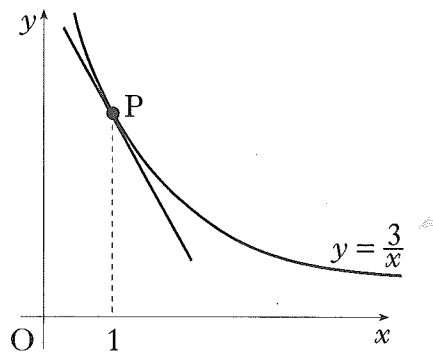
ALL questions should be attempted.

Marks

1. (a) Find the equation of the straight line through the points A(-1, 5) and B(3, 1). 2  
 (b) Find the size of the angle which AB makes with the positive direction of the  $x$ -axis. 2

2. (a) If  $u = \begin{pmatrix} 1 \\ 7 \\ -2 \end{pmatrix}$  and  $v = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ , write down the components of  $u + 3v$  and  $u - 3v$ . 2  
 (b) Hence, or otherwise, show that  $u + 3v$  and  $u - 3v$  are perpendicular. 2

3. Find the equation of the tangent to the curve with equation  $y = \frac{3}{x}$  at the point P where  $x = 1$ .

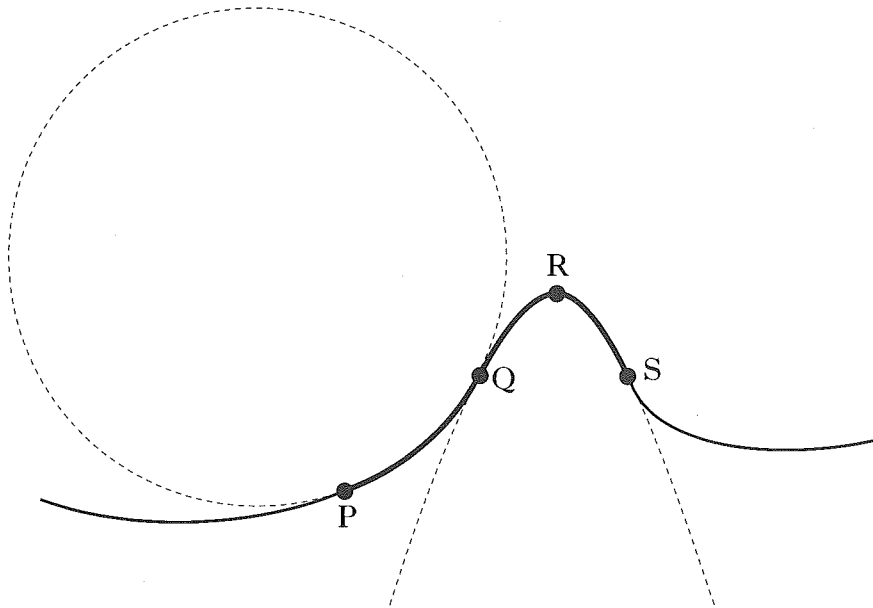


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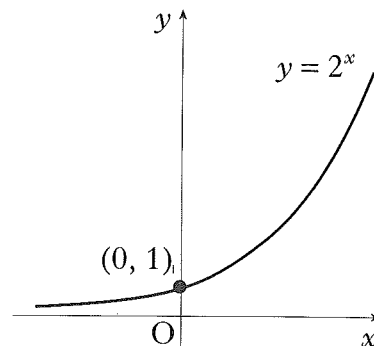
4. (a) Write down the exact values of  $\sin\left(\frac{\pi}{3}\right)$  and  $\cos\left(\frac{\pi}{3}\right)$ . 1  
 (b) If  $\tan x = 4 \sin\left(\frac{\pi}{3}\right) \cos\left(\frac{\pi}{3}\right)$ , find the exact values of  $x$  for  $0 \leq x \leq 2\pi$ . 2
5. Given that  $(x - 2)$  and  $(x + 3)$  are factors of  $f(x)$  where  $f(x) = 3x^3 + 2x^2 + cx + d$ , find the values of  $c$  and  $d$ . 5

[Turn over

6. The side view of part of a roller coaster ride is shown by the path PQRS. The curve PQ is an arc of the circle with equation  $x^2 + y^2 + 4x - 10y + 9 = 0$ . The curve QRS is part of the parabola with equation  $y = -x^2 + 6x - 5$ . The point Q has coordinates (2, 3).



- (a) Find the equation of the tangent to the circle at Q. 4
- (b) Show that this tangent to the circle at Q is also the tangent to the parabola at Q. 2
7. Find  $\int \left( \sqrt[3]{x} - \frac{1}{\sqrt{x}} \right) dx$ . 4
8. The diagram shows part of the graph of  $y = 2^x$ .
- (a) Sketch the graph of  $y = 2^{-x} - 8$ . 2
- (b) Find the coordinates of the points where it crosses the  $x$  and  $y$  axes. 2

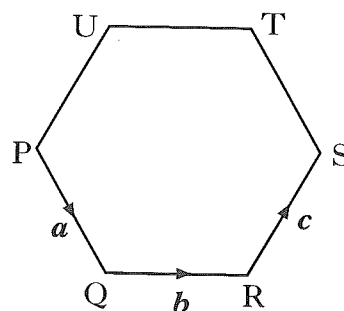


Marks

9. The function  $f$ , defined on a suitable domain, is given by  $f(x) = \frac{3}{x+1}$ .
- (a) Find an expression for  $h(x)$  where  $h(x) = f(f(x))$ , giving your answer as a fraction in its simplest form. 3
- (b) Describe any restriction on the domain of  $h$ . 1

10. A function  $f$  is defined by  $f(x) = 2x + 3 + \frac{18}{x-4}$ ,  $x \neq 4$ .  
Find the values of  $x$  for which the function is increasing. 5

11. PQRSTU is a regular hexagon of side 2 units.  
 $\vec{PQ}$ ,  $\vec{QR}$  and  $\vec{RS}$  represent vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  respectively.  
Find the value of  $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$ .



3

12. If  $\log_a p = \cos^2 x$  and  $\log_a r = \sin^2 x$ , show that  $pr = a$ . 3

[END OF QUESTION PAPER]